Ph.D. Preliminary Examination
Topology
January 18, 1993

Do as many as possible

1. For $i = 1, 2, 3, \ldots$, let X_i be a finite discrete topological space. Find and prove a necessary and sufficient condition so that $\prod_{i=1}^{\infty} X_i$, with the product topology, is discrete.

2. Let (X, d) be a compact metric space with the property that $\forall t < 1$, there are points x_t, y_t so that $d(x_t, y_t) = t$. Prove there are points x and y so that $d(x, y) = 1$.

3. Let $p : X \to Y$ be a covering map where Y is T_1. Prove that if A is a compact subset of X then $\text{card}(p^{-1}(y) \cap A)$ is finite for all $y \in Y$.

4. Let X be compact, T_2 and let A be closed in X. Prove that the quotient space X/A is T_2.

5. Let A be any countable subset of R^2. Prove that $R^2 - A$ is a path connected subspace.

6. Prove that a space X is connected if and only if it has the following property: for every open cover $\mathcal{U}$ of X and every pair U, V of sets in $\mathcal{U}$, there exists a finite chain of sets $U = U_1, \ldots, U_n = V$ of sets in $\mathcal{U}$ such that $U_k \cap U_{k+1} \neq \emptyset$ for $1 \leq k < n$.

7. Let X be normal and let $f : X \to Y$ be continuous, closed and onto. Prove that Y is normal.

8. Let $X =$

   Find $\pi_1(x)$ and the universal cover of X.