University at Albany

Department of Mathematics and Statistics

Preliminary Examination

Mathematical Statistics

January, 2008

Do as many problems as possible!

1. With prior probability 1/4, $\Theta = 2$; otherwise, $\Theta = 1.3$. Draw a random sample of size 3 from a Poisson distribution with parameter Θ. If the 3 observations are 2, 3, and 1, then what is the posterior probability that $\Theta = 2$?

2. A random sample of size 4 is to be drawn from a normal distribution with unknown mean μ and unknown variance $\sigma^2 > 0$. Describe as best you can a test at significance level 0.05 of the hypothesis $\mu = 100$ against the alternative hypothesis $\mu \neq 100$. Call the observed values x_1, x_2, x_3, and x_4. To actually perform the test, you would need a value from a commonly available table; carefully describe how you would find this value if you had the table available.

3. Suppose X_1 and X_2 are independent random variables so that X_1 has $\Gamma(\alpha_1, 1)$ distribution and X_2 has $\Gamma(\alpha_2, 1)$ distribution. Suppose $Y_1 = X_1 + X_2$ and $Y_2 = X_1/(X_1 + X_2)$. Find the joint probability density function of Y_1 and Y_2 and the marginal probability density function of Y_2.
Hint: Recall that if X has $\Gamma(\alpha, \beta)$ distribution, then its probability density function is

$$f(x) = \begin{cases} \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-x/\beta} & \text{if } x > 0 \\ 0 & \text{otherwise.} \end{cases}$$

4. Find the moment generating function for a random variable X with $\Gamma(\alpha, \beta)$ distribution. Use this function to find $E(X)$ and $\mathrm{Var}(X)$.

5. Suppose $X_1, \ldots, X_n$ form a random sample from a distribution given by the probability density function

$$f(x;\theta) = \begin{cases} \theta x^{\theta-1} & \text{if } 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

Describe as best you can the best critical region of size α for testing the hypothesis $H_0 : \theta = 3$ against the alternative hypothesis $H_1 : \theta = 1$.

6. Suppose $X_1, \ldots, X_n$ form a random sample from the uniform distribution on the interval $(0, \theta]$ where $\theta > 0$. Find the maximum likelihood estimator of θ. Justify your answer.

7. A random sample of size 25 is drawn from a normal distribution with unknown mean μ and variance $\sigma^2 = 10$. The observed mean is 8.54. Find a 95% confidence interval for μ. *Hint:* You will need a value b such that $P(|Z| < b) = 0.95$ where Z is a standard normal random variable. You should know approximately the value of b.