University at Albany

Department of Mathematics and Statistics

Preliminary Examination

Mathematical Statistics

June, 2005

Do as many problems as possible!

1. Let $X_1, \ldots, X_n$ denote a random sample from a normal distribution with mean μ and variance 16. Find the value of n so that the 95 percent confidence interval for μ will be $(\bar{x} - 0.98, \bar{x} + 0.98)$.

2.a. Suppose X is a Poisson random variable with mean θ. Find the moment generating function for X.
b. Suppose X and Y are independent Poisson random variables with means θ_X and θ_Y, respectively. What kind of random variable is $X + Y$? Include any relevant parameters, and justify your response.

3. Suppose $X_1, \ldots, X_9$ form a random sample from a distribution which is uniform on $(-1, 1)$. Let $Y_1, \ldots, Y_9$ be the order statistics of this random sample. Find the distribution function of Y_5, and use this to find the probability density function of Y_5.

4. There is an experiment with 3 distinct outcomes A, B, and C. You wish to test the hypothesis $H_0 : P(A) = 0.6, P(B) = 0.3$, and $P(C) = 0.1$ against all other hypotheses. You perform this experiment 500 times. In doing so, you find that A occurs 320 times, B occurs 116 times, and C occurs the remaining times. Carefully describe a test which determines whether you may reject H_0 at the approximate 1 percent significance level. This test involves a value from a commonly available table; since you don't have the table, describe where to find this value and how you would use the value if you had it.

5. Let $X_1, \ldots, X_{100}$ denote a random sample from a distribution with probability density function

$$f(x;\theta) = \begin{cases} 1/\theta & \text{if } 0 < x < \theta \\ 0 & \text{otherwise} \end{cases}$$

where $\theta > 0$ is a parameter.

a. Let $Y = \max(X_1, \ldots, X_{100})$. Is Y a sufficient statistic for θ? Justify your answer.

b. What is the maximum likelihood estimator for θ? Is this estimator unbiased? Justify your answer.

6. Suppose

$$f_1(x) = \begin{cases} c_1 x & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

and

$$f_2(x) = \begin{cases} c_2 x^2 & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

are continuous probability density functions where c_1 and c_2 are constants.

a. Find the constants c_1 and c_2.

b. Let $X_1, \ldots, X_{400}$ denote a random sample from a distribution with probability density function $f(x)$. Describe a best test of the hypothesis $f(x) = f_1(x)$ against the hypothesis $f(x) = f_2(x)$.

7. Let X_1 and X_2 be independent standard normal random variables. Suppose $Y_1 = X_1/X_2$ and $Y_2 = X_2$.

a. Find the joint probability density function of Y_1 and Y_2.

b. Use part a to find the probability density function of Y_1.