University at Albany

Department of Mathematics and Statistics

Preliminary Examination

Mathematical Statistics

August, 2003

Do as many problems as possible!

1. Let $X_1, \ldots, X_n$ denote a random sample of size n from the distribution with probability density function

$$f(x) = \begin{cases} e^{-(x-\theta)} & \text{if } x > \theta \\ 0 & \text{otherwise} \end{cases}$$

where θ is a parameter. Which of the following statistics are sufficient statistics for θ? Justify.

a. $\min(X_1, \ldots, X_n)$

b. $\max(X_1, \ldots, X_n)$

2. Suppose $X_1, \ldots, X_n$ are independent random variables with distributions $N(\mu_1, \sigma_1^2)$, $\ldots$, $N(\mu_n, \sigma_n^2)$, respectively, where $N(\mu, \sigma^2)$ is the normal distribution with mean μ and variance σ^2. Let $k_1, \ldots, k_n$ be real constants. What is the distribution of $Y = k_1X_1 + \ldots + k_nX_n$? Justify your answer by using moment generating functions.

3. Consider the following experiment. Roll an ordinary die, and let A_i be the event the number i comes up. Consider performing this experiment 300 times. Describe a test at the approximate significance level $\alpha = 0.05$ of the hypothesis $H_0 : P(A_i) = 1/6$ for $i = 1, \ldots, 6$ against all other hypotheses. (If you need a value from a widely available table, describe how to find the value.)

4. Let $X_1, \ldots, X_n$ denote a random sample of size n from the normal distribution $N(\theta, 1)$. Find, with proof, the maximum likelihood estimator for θ and determine whether this estimator is unbiased.

5. Let X be a standard normal random variable. Find, with proof, the probability density function of X^2.

6. Suppose $X_1, \ldots, X_n$ are independent uniform random variables on the interval $(0, \theta)$ where $\theta > 0$ is a parameter. Let $Y_n = \max(X_1, \ldots, X_n)$. Find the probability density function of Y_n and the expected value of Y_n.