Classical Algebra Written Assignment No. 3

due Wednesday, November 5, 2003

Directions: Written assignments must be typeset. While it is neither necessary nor desirable to show small details of computation, you must indicate what you are doing and explain any reasoning used. Accuracy is important in this assignment.

If you are in the writing intensive division of the course, you must complete each written assignment in a satisfactory way. This may require re-submission, possibly more than once, after the initial evaluation.

1. Find the order of $[19]_{107}$ in $\mathbb{Z}/107\mathbb{Z}$.

2. Find the least non-negative residue of $31^{1777} \pmod{113}$.

3. Find all integers $x$ that satisfy the following simultaneous congruences:

   \[
   x \equiv 4 \pmod{11} \\
   x \equiv 1 \pmod{8} \\
   x \equiv 2 \pmod{15}
   \]

4. Encrypt the length 9 string “Sell out!” one character at a time by forming the sequence consisting of the 9 least non-negative residues modulo 10001 of the $7^{th}$ powers of the ASCII codes, which are numbers from 32 to 127.

5. What technique should be used to reverse the encryption of the ASCII codes in the previous problem?