Due Wednesday, October 17

1. Let $f$ be a linear map from $\mathbb{R}^3$ to $\mathbb{R}^3$ for which

   (a) $f(1, 0, 0) = (1, 2, 3)$.
   (b) $f(0, 1/2, 0) = (3, 2, 1)$.
   (c) $f(-1, 0, 2) = (4, -6, 2)$.

   Find all possible $3 \times 3$ matrices $A$ for which the formula $f(x) = Ax$ is valid for all $x$ in $\mathbb{R}^3$.

   Hint: Use the rules for abstract linearity to work out what happens under $f$ to $(0, 1, 0)$ and $(0, 0, 1)$.

2. Let $g$ be the linear map from $\mathbb{R}^4$ to $\mathbb{R}^4$ that is defined by $g(x) = Bx$ where $B$ is the matrix

   $\begin{pmatrix}
   1 & 2 & -4 & 3 \\
   -2 & -1 & -1 & 5 \\
   1 & 3 & 2 & -1 \\
   1 & 1 & -1 & -1
   \end{pmatrix}$.

   Find a $4 \times 4$ matrix $C$ for which the linear map $h$ given by multiplication by $C$ has the property that both $h(g(x)) = x$ and $g(h(y)) = y$ for all $x$ and all $y$ in $\mathbb{R}^4$.

3. Could the previous exercise have been completed successfully if the given matrix $B$ had been one of the matrices appearing in the assignment due Friday, October 12\(^1\)?

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\(^1\)URI: http://math.albany.edu:8000/math/pers/hammond/course/mat220/assgt/la011015.html