**TOPOLOGY PRELIMINARY EXAMINATION**

**June 11, 1999**

1. Define the Lebesgue number of a covering of a metric space $X$. Prove that if $X$ is compact, then every covering of $X$ has a positive Lebesgue number.

2. Prove that a compact Hausdorff space is normal.

3. Let $X$ be a second countable space. Let A be a subset of $X$ so that the subspace topology on $A$ is discrete. Prove that $A$ is countable.

4. Prove that $\mathbf{R}/\mathbf{Q}$ is not Hausdorff.

5. Show that the product of finitely many connected spaces is connected.

6. Show that the Cantor (middle third) set is a closed, perfect, totally disconnected subset of $[0, 1]$. (Recall that a set is perfect if every point is an accumulation point.) Conversely, show that every such set is homeomorphic to the Cantor set.

7. (a.) Prove that $\Pi_1(S^1) = Z$. You may quote results from covering space theory in your proof.
   (b.) Show there is no retraction of $D^2$ onto $S^1$.

8. Use covering space theory to prove that the fundamental group of the figure eight is not abelian.