Topology Preliminary Examination

September 6, 1996

Do as many problems as possible.

1. Suppose X and Y are compact, T^2 spaces and $f : X \to Y$ is a bijection (not assumed to be continuous) such that if C is a compact set in X then $f(C)$ is compact in Y. Prove that f is a homeomorphism.

2. Let A_j for j in J be a set in X. Suppose that $U\ \overline{A_j}$ is closed. Prove that $\overline{U\ A_j} = U\ \overline{A_j}$.

3. Let T be the usual topology on R and let V be the topology on R where the open set are of the form (a, ∞) for a in R together with the empty set and R. Is $R \times R$ with the product topology $T \times Y$ regular?

4. Prove that $X \times Y$ is connected if and only if X and Y are connected.

5. Let X be a compact metric space with metric d and let A be the family of closed sets in A. Define a function on $A \times A$ by $e(a, b) = \inf\{r : a \subset N_r(b)$ and $b \subset N_r(a)$ where $N_r(c)$ is the set of points in X whose distance to c is less than r.
   Prove that e defines a metric on A.

6. Let $p : E \to B$ be a covering map where B is connected. Suppose that $p^{-1}(b)$ is a set with n-members. Prove that $p^{-1}(b)$ is a set with n-members for each $b \in B$.

7. Prove that π_1 is not abelian.

8. Define a relation $\sim$ on R^2 by $(x_1, y_1) \sim (x_2, y_2)$ provided $x_1 + y_1^2 = x_2 + y_2^2$.
   (a) Prove that $\sim$ is an equivalence relation.
   (b) Describe the identification (quotient) space $R^2/\sim$.