Ph.D. Preliminary Exam

Topology

June 8, 1993

Do as many as possible.

1. Prove that every metric space is normal.

2. Prove that X is path connected if and only if X is connected and every point has a path connected open neighborhood.

3. Let X be a metric space in which every covering has a countable subcovering. Prove that X is second countable.

4. Prove that $\mathbf{R}^3$ - 2 points is simply connected.

5. Prove whether $\mathbf{R}^1$ with the finite complement topology has any of the following properties.

   compact   Hausdorff   first countable

6. Let X be normal and let $f : X \to Y$ be an open and closed quotient map. Prove that Y is normal.

7. Let X be compact and Hausdorff. Prove that if $f : X \to X$ is any continuous function then there is a non-empty subset A so that $f(A) = A$.

8. Prove that if $p : X \to Y$ is a surjective covering projection then p is a quotient map.