Topology Preliminary Exam

August 25, 2005

Do as many problems as possible.

1. Suppose that X is metrizable. Prove that X is 2^{nd} countable if and only if X contains a countable dense subset.

2. Let $\mathbf{R}^1$ be the real line and let Z be the integers. Prove or disprove that $\mathbf{R}/\mathbf{Z}$ is compact.

3. Show that $\mathbf{R}^2$ minus a countable set is path connected. Hint: look at the set of lines through a given point.

4.Let $p : E \to B$ be a covering map. Suppose that f and g are continuous functions from I, the closed unit interval, to E such that

(1) $pf = pg$

(2) $f(0) \neq g(0)$.

Prove that for all t in I, $f(t) \neq g(t)$

5. Let $X = RP^2 \vee S^1$, the one-point union of the projective plane and a circle.

a.Compute the fundamental group of X

b. Find all 2-sheeted and 3-sheeted coverings of X.

c. Find the universal cover of X.

6. Let X be a topological space

a. Suppose that X has a finite number of components. Prove that each component is open.

b. Give an example to show that the conclusion is false if X has an infinite number of components.

7. Prove that a compact Hausdorff space is normal

8. For each natural number n let $X_n = \{0, 1\}$ where $\{0, 1\}$ is given the discrete topology. Let X be the product of the spaces X_n with the product topology

Let A be the subset of X with the property that if $a \in A$ and $a = \{a_1, a_2, ...\}$ then there exists an n such that $0 = a_n = a_{n+2} = a_{n+4} = ...$ and $1 = a_{n+1} = a_{n+3} = a_{n+5} = ...$ What is the closure of A?