

Topology Prelim

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Do as many problems as possible.

1. Let X be a connected metric space with metric d . Suppose that $d(a, b) = 1$, prove there exists a point c in X such that $d(a, c) = 1/2$.
2. Let $X = \prod_{i=1}^{\infty} \{0, 1\}$ with the product topology where $\{0, 1\}$ is the two point set with the discrete topology. Find the closure with explanation of the following sets
 - a. $\{x | x_i = 0 \text{ for finitely many } i\}$
 - b. $(0, 0, \dots), (1, 0, 0, \dots), (1, 1, 0, 0, \dots), (1, 1, 1, 0, 0, \dots), \dots (1, 1, \dots, 1, 0, 0, \dots), \dots$
3. Let $f : X \rightarrow Y$ be a covering map.
 - a. Prove that f is open
 - b. Prove that X is compact if Y is compact and if f is an n fold covering where n is finite.
4. Let X be the following subset of R^2 $\{(x, y) : x^2 + y^2 = 1\} \cup \{(x, 0) : -1 < x < 1\}$.
 - a. What is $\prod_1(X)$?
 - b. Find all 2 fold coverings of X .
5. A space X is said to be completely regular if it is Hausdorff and given any point x and disjoint closed set A there is a continuous function from X to $[0, 1]$ such that $f(x) = 1$ and f is 0 on A . Prove that the product of two completely regular spaces is completely regular.
6. Let $[0, 1]$ be given the half open interval topology, where the basis of open sets consists intervals closed on the left but open on the right. Prove or disprove that this space is compact.
7. Let X be a compact subspace of R^2 . Prove that every continuous real valued function defined on X is bounded.
8. Let X be a metric space with metric d . Let $x \in X$ and let A be a subset of X and define $d(x, A) = \inf\{d(x, a) | a \text{ is in } A\}$. Prove that $d(x, A) = d(x, cl(A))$ where $cl(A)$ is the closure of A .