University at Albany

Department of Mathematics and Statistics

Preliminary Examination

Mathematical Statistics

June, 2006

Do as many problems as possible!

1. Let $X_1, \ldots, X_n$ denote a random sample from a normal distribution with mean μ and variance 9. Find the value of n such that the 95 percent confidence interval for μ will be $(\bar{x} - 0.196, \bar{x} + 0.196)$. Also find the value of n so that the 95 percent confidence interval for μ will be $(\bar{x} - 0.098, \bar{x} + 0.098)$.

2. Suppose

$$f_1(x) = \begin{cases} c_1(1-x) & \text{if } -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

and

$$f_2(x) = \begin{cases} c_2(1+x) & \text{if } -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

are continuous probability density functions where c_1 and c_2 are constants.

a. Find the constants c_1 and c_2.

b. Let $X_1, \ldots, X_{100}$ denote a random sample from a distribution with probability density function $f(x)$. Describe a best test of the hypothesis $f(x) = f_1(x)$ against the hypothesis $f(x) = f_2(x)$.

3. Suppose W is a standard normal random variable and V is a chi-square random variable with 10 degrees of freedom. Suppose also that W and V are independent; thus the joint probability density function of W and V is

$$h(w, v) = \begin{cases} \frac{1}{\sqrt{2\pi}} e^{-w^2/2} \frac{1}{\Gamma(5) 2^5} v^4 e^{-v/2} & \text{if } v > 0 \\ 0 & \text{otherwise.} \end{cases}$$

Let $T = W/\sqrt{V/10}$ and $U = V$. Find the joint probability density function of T and U, and use this result to find the probability density function of T.

4. In a sample of size 10 from a normal distribution with unknown mean μ and unknown variance, the observed values have sample mean 2.38 and sample variance 4.23. Describe how to find a 99 percent confidence interval for μ; this description involves a value b from a commonly available table. You need not give its value, but you should describe which table you would use and how you would find the value within the table.

5. Let $X_1, \ldots, X_n$ be a random sample from the normal distribution $N(0, \theta)$ where $0 < \theta < \infty$. Is $\sum_{i=1}^{n} X_i^2$ a sufficient statistic for θ? Justify.

6. Let $X_1, \ldots, X_{2006}$ be i.i.d. random variables which are uniform on the interval $(0, 2)$. Let $Y_1, \ldots, Y_{2006}$ be the order statistics of this random sample. Find, with justification, the distribution function for Y_{2006}, and use this function to find the probability density function of Y_{2006}. Then find the expected value of Y_{2006}.

7. Suppose $X_1, \ldots, X_n$ represents a random sample from a distribution with probability density function

$$f(x; \theta) = \begin{cases} 1/\theta & \text{if } 0 < x \leq \theta \text{ and } 0 < \theta < \infty \\ 0 & \text{otherwise.} \end{cases}$$

Find the maximum likelihood estimator of θ. Is it unbiased? Is it consistent? Justify!