University at Albany

Department of Mathematics and Statistics

Preliminary Examination

Mathematical Statistics

August, 2004

Do as many problems as possible!

1. Let

$$f(x;\theta) = \begin{cases} 1/\theta & \text{if } 0 < x \leq \theta \\ 0 & \text{otherwise} \end{cases}$$

be a continuous probability density function depending on a parameter $\theta > 0$. Let $X_1, \ldots, X_n$ denote a random sample of size n from this distribution. Find, with proof, the maximum likelihood estimator of θ and the expectation of this maximum likelihood estimator. Also, is this maximum likelihood estimator unbiased and is this maximum likelihood estimator consistent?

2.a. Suppose X is a normal random variable with mean 0 and variance σ^2. Find the moment generating function of X.

b. Suppose X and Y are independent normal random variables with variances σ_X^2 and σ_Y^2. Suppose X and Y each have mean 0. What kind of random variable is $X + Y$? Include any relevant parameters, and justify your response.

3. Suppose

$$f_1(x) = \begin{cases} c_1 x^2 & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

and

$$f_2(x) = \begin{cases} c_2(1 - x^2) & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

are continuous probability density functions where c_1 and c_2 are constants.

a. Find the constants c_1 and c_2.

b. Let $X_1, \ldots, X_{100}$ be a random sample from a distribution with probability density function $f(x)$. Describe a best test of the hypothesis $f(x) = f_1(x)$ against the hypothesis $f(x) = f_2(x)$.

4. Let $X_1, \ldots, X_{10}$ be a random sample from a normal distribution with mean μ and unknown variance σ^2. Describe a test at significance level 0.05 of the hypothesis $\mu = 0$ against the alternative $\mu \neq 0$ if the observed values are 3.8, -1.2, 4.0, 2.3, -0.8, 1.5, -1.1, 2.5, -0.4, and 0.1. (You need not perform tedious calculations but should describe them. To actually perform the computations, you would need a value from a commonly available table. You need not give the value itself, but you should describe where you would find the value if you had the table available.)

5. Let $X_1, \ldots, X_n$ denote a random sample of size $n > 1$ from a distribution with probability density function

$$f(x, \theta) = \begin{cases} e^{-(x-\theta)} & \text{if } x > \theta \\ 0 & \text{otherwise.} \end{cases}$$

Let $Y_1 = \min(X_1, \ldots, X_n)$.

a. Find, with justification, the probability density function of Y_1.

b. Is Y_1 a sufficient statistic for θ? Justify your answer.

6. Let $X_1, \ldots, X_n$ denote a random sample from a normal distribution with mean μ and variance 100. Find the value of n so that the 95 percent confidence interval for μ will be $(\bar{x} - 1.96, \bar{x} + 1.96)$. Also find the value of n so that the 95 percent confidence interval for μ will be $(\bar{x} - 0.98, \bar{x} + 0.98)$. (Recall: If Z is a standard normal random variable, then $P(|Z| < 1.96) = 0.95$.)