University at Albany

Department of Mathematics and Statistics

Preliminary Examination

Mathematical Statistics

January, 2004

Do as many problems as possible!

1. Suppose $X_1, \ldots, X_{200}$ form a random sample from a distribution which is uniform on $(0, 1)$. Let $Y_1, \ldots, Y_{200}$ be the order statistics of this random sample. Find the distribution function of Y_1, and use this to find the probability density function of Y_1. Then find the expected value of Y_1.

2. Let $X_1, \ldots, X_n$ represent a random sample from a distribution with probability density function

$$f(x; \theta) = \begin{cases} \theta x^{\theta-1} & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

where $\theta > 0$ is a parameter. Find the maximum likelihood estimator of θ.

3. There is an experiment with 4 distinct outcomes A, B, C, and D. You wish to test the hypothesis $H_0 : P(A) = 0.50, P(B) = 0.30, P(C) = 0.15, P(D) = 0.05$ against all other hypotheses. You perform this experiment 1000 times. In doing so, you find that A occurs 453 times, B occurs 320 times, C occurs 142 times, and D occurs the remaining times. Carefully describe a test which determines whether you may reject H_0 at the approximate 1 percent significance level. This test needs a value from a commonly available table; since you don't have the table, describe where to find this value and how you would use it.

4. Let X be a standard normal distribution. What kind of distribution does X^2 have? Prove your answer.

5. a. Is the sum of the observations of a random sample of size n from a Poisson distribution with parameter $\theta > 0$ a sufficient statistic for θ? Justify.

b. Consider the maximum of the observations of a random sample of size $n > 1$ from a distribution with probability distribution function

$$f(x;\theta) = \begin{cases} e^{-(x-\theta)} & \text{if } x > \theta \\ 0 & \text{otherwise} \end{cases}$$

with real parameter θ. Is this maximum a sufficient statistic for θ? Justify.

6. You have a random sample of size n from a normal distribution with unknown mean θ and variance 100. You want to find n large enough so that the length of the confidence interval (from left endpoint to right endpoint) is at most 0.196. Find such a value of n so that n is as small as possible. If you instead were willing to have a confidence interval with twice this length, what would you need to do to n? (Note: All confidence intervals in this problem are 95 percent confidence intervals.)