COMPLEX ANALYSIS PRELIM. (AUGUST 25, 2005)

1. Find an analytic function $f(z)$ whose real part is

$$Re(f(z)) = xy + 3, \ (z = x + iy).$$

Does such a function exist? Justify your answer.

2. Let u be a real-valued harmonic function. For what functions f is the function $f(u)$ harmonic?

3. Let f be analytic in a domain Ω and $Re(f)$ be a constant on Ω. Show that f is a constant.

4. Construct a conformal mapping of $\mathbb{C} \setminus ([-1, 0] \cup [-i, i])$ onto the init disk.

5. State and prove Morera's Theorem.

6. Compute the integral

$$\int_{|z|=1/2} \frac{dz}{(2z - \bar{z})^8}.$$

7. How many roots of the equation $z^4 - 6z + 3 = 0$ have their modulus between 1 and2?

8. Let f be an entire function whose modulus is constant on some circle. Prove that $f(z) = C(z - z_0)^n$.

9. By Picard's Theorem every meromorphic function has at most 2 exceptional values (that is there are at most two complex numbers which are

not in the range). How many exceptional values does $\tan z$ have? Find them.

10. Prove that there exists a constant C such that for every polynomial P

$$\left| \int_{-1/2}^{1/2} P(x)dx \right| \le C \int_{|z|=1} |P(z)||dz|.$$