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-0400 Subject: (Fwd) C-Prelim Tex File Priority: normal  
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13:18:17 -0400 (EDT) To: richardg@csc.albany.edu Subject: C-Prelim Tex File

### Prelim in Complex Analysis, June 2001

1. Evaluate the following integrals.

$$\int_{|z|=2} \tan z \, dz, \quad \int_0^\pi \frac{dt}{5 - 4 \cos t}.$$

2. Find the Laurent series of the function

$$f(z) = \frac{1}{(z-1)(z-2)}$$

in the region  $1 < |z-3| < 2$ .

3. Suppose  $f(z)$  is an entire function with  $\operatorname{Re} f(z) > 10$  for all  $z$ . Show that  $f$  is constant.  
4. Let  $F$  be the family of functions  $f$  analytic in  $|z| < 1$  such that

$$\int_{|z|<1} |f(z)| \, dA(z) \leq 1,$$

where  $dA$  is area measure on  $|z| < 1$ . Show that  $F$  is a normal family.

5. Does there exist an analytic function  $f$  in  $|z| < 1$  such that

$$0 < \left| f\left(\frac{1}{n}\right) \right| < e^{-n}$$

for  $n = 2, 3, 4, \dots$ ? Justify your answer.

- 6.

- (a) Show that

$$\left| \frac{1-2z}{2-z} \right| < 1$$

for all  $|z| < 1$ .

- (b) Suppose  $f$  is analytic in  $|z| < 1$ ,  $f(0.5) = 0$ , and  $|f(z)| \leq 1$  for all  $|z| < 1$ . Show that

$$|f(z)| \leq \left| \frac{1-2z}{2-z} \right|$$

for all  $|z| < 1$ .

END