1. For which primes $p$ are the Sylow $p$-subgroups of the symmetric group $S_4$ normal?

2. If $G$ is any group, $H$ a subgroup, and $x, y$ elements of $G$, show that $xH = Hy$ if and only if $x$ and $y$ both belong to the normalizer $N_G(H)$ and determine the same element of $N_G(H)/H$.

3. Show that the group $\text{SL}_2(\mathbb{F}_3)$ has a normal subgroup of order 8. List the 8 elements of this subgroup, and explain why this group of order 8 is not isomorphic to the dihedral group $D_4$.

4. Let $G$ be a finite group and $H$ a subgroup of index 3 in $G$ that is not a normal subgroup of $G$. Show that $H$ contains a subgroup $N$ that is normal in $G$ for which $G/N \cong S_3$.

5. Find an explicit list of groups that represent all isomorphism classes of groups of order 66.