Math 502

Written Assignment No. 2

due Wednesday, March 8, 2006

Directions: Use *Maple* for assistance in responding to the following problems. Please typeset your solutions. Explain what you have done. *Maple* session details are not necessary unless you think it important to include them. Accuracy is important.

Although you may refer to books and notes, you may not seek help from others on this written assignment.

Answer the following questions:

1. What is the length of the repeating pattern in the decimal expansion of

$$\frac{1}{1 + 2^{100}} \ ?$$

   (Don't even *think* about trying to compute the expansion.)

2. If $P = a + bx + cx^2 + dx^3 + ex^4 + fx^5$ is a polynomial, find polynomials $q_0, q_1, q_2, \ldots$ of degree at most 1 such that

$$P = \sum_{j \geq 0} q_j (x^2 + 1)^j \quad .$$

3. Factor the polynomial $x^{31} - 1$ as

   (a) a rational polynomial.
   (b) a polynomial mod 2.
   (c) a polynomial mod 3.
   (d) a polynomial mod 5.

4. Find the partial fraction expansion of

$$\frac{1}{x^{31} - 1}$$

   regarded as a "rational function" with coefficients mod 2.

5. The sequence of integers

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   [61, 13, 25, 37, 13, 28, 17, 81, 40, 3, 75, 91, 13, 25, 20, 37, 81,
    25, 91, 95, 25, 91, 17, 70, 37, 17, 25, 77, 25, 75, 85, 25, 13, 91,
    3, 25, 19, 75, 20, 37, 19, 81, 25, 3, 91, 25, 23, 37, 25, 73, 37,
    17, 81, 25, 85, 37, 28, 59, 17, 37, 6]
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   is the sequence of 17-th powers mod 97 of numbers obtained from a sequence of ASCII codes by left-shifting each code 30 units so the sequence occupies the interval $[2, 96]$ instead of the normal printable ASCII interval $[32, 126]$. What text string led to the original sequence?