Four Kinds of Isometries of the Cartesian plane

- Recall that by definition an isometry of \( \mathbb{R}^n \) is a distance-preserving affine transformation. Please refer to the discussion accompanying the assignment due February 4 as well as to exercises involving isometries of \( \mathbb{R}^2 \) posed in recent assignments.

- Four types of isometries of \( \mathbb{R}^2 \) are (a) rotations, (b) translations, (c) reflections, and (d) glide reflections.

- A rotation has a point called its center. Under a rotation every point is moved through a fixed angle around the circle with the given center on which it lies. A rotation is orientation-preserving.

- A translation is an affine transformation of the form \( x \mapsto x + v \). A translation is orientation-preserving.

- A reflection has a line called its axis. Under a reflection every point is sent to its mirror image relative to the axis. The line segment from a point to its image under the reflection is perpendicularly bisected by the axis. A reflection is orientation-reversing.

- A glide reflection is the transformation that results when a reflection is followed with the translation by a non-zero vector parallel to the axis of the reflection. A glide reflection is an orientation-reversing isometry with no fixed point.

- The identity transformation may be regarded as both a translation and a rotation, but is sometimes regarded as neither.

Exercises due Friday, February 13

Let \( f \) be the isometry of \( \mathbb{R}^2 \) that is obtained by following rotation about the origin counterclockwise through the angle \( \pi/6 \) with translation by the vector \( (2, 0) \).

1. Find a \( 2 \times 2 \) matrix \( U \) and a vector \( v \) in the plane such that \( f(x) = Ux + v \) for each \( x \) in \( \mathbb{R}^2 \).

2. Find a point \( c \) in \( \mathbb{R}^2 \) such that \( f(c) = c \).

3. It being claimed without proof for the moment that every isometry of \( \mathbb{R}^2 \) falls into one of the four classes enumerated above, explain from that why this \( f \) must be a rotation.

4. Give a geometric construction of the center of rotation for \( f \).

5. What is the angle of rotation for \( f \)?