Discussion

- A **linear combination** of points or vectors $v_1, \ldots, v_N$ is any point of the form
  \[
  \sum_j c_j v_j
  \]
  where the $c_j$ are numbers. The numbers $c_j$ are called the **coefficients** of the linear combination of the given points or vectors.

- **Definition:** The sum
  \[
  \sum_j c_j
  \]
  of the coefficients in a linear combination is called the **weight** of the linear combination.

- A **barycentric combination** of points or vectors $v_1, \ldots, v_N$ is any weight 1 linear combination of them in which each coefficient is non-negative.

- If $A$ and $B$ are two different points of the plane (or of space or of $n$-dimensional space), then the line determined by $A$ and $B$ is the set of all weight 1 linear combinations of $A$ and $B$, and the line segment between $A$ and $B$ is the set of all barycentric combinations of $A$ and $B$. Note that if $V = B - A$ is the vector from $A$ to $B$, then the line determined by $A$ and $B$ is the set of all points $A + tV$, and the line segment $AB$ is the subset of these points with $0 \leq t \leq 1$.

**Theorem.** If $A$, $B$, and $C$ are any non-collinear points in the Cartesian plane, then every point $X$ of the plane is a unique weight 1 combination of $A$, $B$, and $C$.

- If $A$, $B$, and $C$ are any non-collinear points in the Cartesian plane, then a point $X$ in the plane lies in the triangle determined by the three points if and only if it is a barycentric combination of $A$, $B$, and $C$.

Exercises due Friday, January 23

Let $A$, $B$, $C$, and $D$ be the points in the Cartesian plane that are given by

$A = (0, -1)$, $B = (3, 4)$, $C = (-1, 1)$, and $D = (1, 2)$,

and let $T$ be the triangle with vertices $A$, $B$, and $C$.

1. Find the midpoint of the line segment $AB$.
2. For which values of $t$ does the point $(1 - t)A + tB$ lie on the line segment $AB$?
3. Find the point where the line $AC$ meets the line $BD$. Does this intersection point lie on both of the line segments $AC$ and $BD$?
4. Find the point where the three medians of $T$ meet.
5. Find the point where the three perpendicular bisectors of the sides of $T$ meet.
6. Find the barycentric coordinates of the point $(2, 2)$ with respect to the vertices of the triangle $T$.
7. How much information about the topic of barycentric coordinates can you find on the world wide web?