Written Assignment No. 3

due November 11, 2005

General Directions: Written assignments should be submitted typeset. What you submit must represent your own work.

Assigned Exercises

Read these directions carefully: For each of the following statements either provide a proof that the statement is true or label the statement as false and provide justification.

1. If \( \mathbb{Z} \) denotes the ring of integers and \( \mathbb{R} \) denotes the field of real numbers, then

\[
\left\{ a + b\sqrt{2} \in \mathbb{R} \mid a, b \in \mathbb{Q} \right\}
\]

is a subring of \( \mathbb{R} \).

2. If \( F \) is a finite field with \(|F| = q\) and \( F[t] \) denotes the ring of polynomials with coefficients in \( F \), then the number of elements in the ring

\[
A = F[t]/(t^q - t)F[t]
\]

(of all congruence classes of polynomials modulo the polynomial \( t^q - t \)) is given by

\[
|A| = q^q.
\]