Classical Algebra

Written Assignment No. 3

due Thursday, October 30, 2008

Directions

Written assignments must be typeset.

While it is neither necessary nor desirable to show small details of computation, you must indicate what you are doing, give major steps in computation, and explain any reasoning used.

Accuracy is important. With 5 problems in an assignment worth 10 points, there is limited room for partial credit on a problem.

Problems

1. Find the order of $[5]_{59}$ in $\mathbf{Z}/59\mathbf{Z}$.
2. Find the least non-negative residue of $41^{137} \pmod{2503}$.
3. Find the 8-adic "decimal" expansion of the rational number
$$\frac{56381}{4088}\quad .$$
4. Characterize all integers x that satisfy the following simultaneous congruences:
$$\begin{aligned} x &\equiv 4 \pmod{11} \\ x &\equiv 3 \pmod{8} \\ x &\equiv 5 \pmod{15} \end{aligned}$$
5. Prove the following:

Proposition. *If $m > 1$ is an integer that is the product of **distinct** primes $p_1, \ldots, p_r$, and e denotes the least common multiple of the integers $p_1 - 1, \ldots, p_r - 1$, then the order of any unit in $\mathbf{Z}/m\mathbf{Z}$ must divide e.*