Classical Algebra

Written Assignment No. 4

due Friday, November 9, 2007

Directions

Written assignments must be typeset.

While it is neither necessary nor desirable to show small details of computation, you must indicate what you are doing, give major steps in computation, and explain any reasoning used.

Accuracy is important. With 5 problems in an assignment worth 10 points, there is limited room for partial credit on a problem.

Please remember that no collaboration is permitted on this assignment.

Problems

1. Find the order mod 43 of
   (a) 2.
   (b) 3.
   (c) 7.
2. Find the smallest integer $u > 1$ such that for every integer x one has
$$x^{5u} \equiv x \pmod{1591}$$
3. Find the smallest positive integer that has order 190 modulo 191. (Note that 191 is prime.)
4. Find the quotient and remainder when the polynomial $x^7 - 1$ is divided by the polynomial $x^3 - x - 1$ and these polynomials are regarded as having coefficients that are
   (a) rational numbers.
   (b) integers modulo 2.
5. Let a and m be integers with $m \geq 2$.
   (a) Give an example of an integer $a > 0$ that is primitive modulo 6.
   (b) Find the smallest integer $a > 0$ that is primitive modulo 17.
   (c) Prove that if $m = p_1 p_2 \ldots p_r$ is the product of r distinct primes with $r \geq 2$ and each $p_j > 2$, then there is no integer a that is primitive modulo m.