Classical Algebra

Written Assignment No. 4

due Tuesday, November 21, 2006

Directions

Written assignments must be typeset.

While it is neither necessary nor desirable to show small details of computation, you must indicate what you are doing, give major steps in computation, and explain any reasoning used.

Accuracy is important. With 5 problems in an assignment worth 5 points, there will be no room for partial credit on a problem.

If you are in the writing intensive division of the course, you must complete each written assignment in a satisfactory way. This may require re-submission, possibly more than once, after the initial evaluation.

Please remember that no collaboration is permitted on this assignment.

Problems

1. Find the order mod 43 of
   (a) 2.
   (b) 3.
   (c) 7.
2. Find the smallest integer $u > 1$ such that for every integer x one has
$$x^{5u} \equiv x \pmod{1591}$$
3. Find the smallest positive integer that has order 190 modulo 191. (Note that 191 is prime.)
4. Find the quotient and remainder when the polynomial $x^7 - 1$ is divided by the polynomial $x^3 - x - 1$ and these polynomials are regarded as having coefficients that are
   (a) rational numbers.
   (b) integers modulo 2.
5. Let a and m be integers with $m \geq 2$.
   (a) State what it means, by definition, for a to be *primitive* modulo m.
   (b) Give an example of an integer $a > 0$ that is primitive modulo 6.
   (c) Prove that if $m = p_1 p_2 \ldots p_r$ is the product of r distinct primes with $r \geq 2$ and each $p_j > 2$, then there is no integer a that is primitive modulo m.