Classical Algebra Written Assignment No. 3

due Monday, November 11, 2002

Directions: Written assignments must be typeset. While it is neither necessary nor desirable to show small details of computation, you must indicate what you are doing and explain any reasoning used. Accuracy is important; with 5 problems in an assignment worth 5 points, there will be no room for partial credit on a problem.

If you are in the writing intensive division of the course, you must complete each written assignment in a satisfactory way. This may require re-submission after an initial evaluation.

1. Find the order of $[23]_{59}$ in $\mathbb{Z}/59\mathbb{Z}$.

2. Find the least non-negative residue of $29^{225} \pmod{257}$.

3. Find all integers $x$ that satisfy the following simultaneous congruences:

$$
\begin{align*}
  x &\equiv 7 \pmod{11} \\
  x &\equiv 6 \pmod{8} \\
  x &\equiv 10 \pmod{15}
\end{align*}
$$

4. Encrypt the length 9 string “Sell out!” one character at a time by forming the sequence consisting of the 9 least non-negative residues modulo 10001 of the $7^{th}$ powers of the ASCII codes, which are numbers from 32 to 127.

5. What technique should be used to reverse the encryption of the ASCII codes in the previous problem?