Linear Algebra (Math 220)
Assignment due Tuesday, March 11

Midterm Test: Tuesday, March 18

1 Preparation

Expect a quiz.

Suggested Reading:

- Lay § 4.7
- Hefferon §§ 3.IV – 3.V

2 Exercises

1. Let g be the linear map from $\mathbf{R}^4$ to $\mathbf{R}^4$ that is defined by $g(x) = Bx$ where B is the matrix
$$\begin{pmatrix} 1 & 2 & -4 & 3 \\ -2 & -1 & -1 & 5 \\ 1 & 3 & 2 & -1 \\ 1 & 1 & -1 & -1 \end{pmatrix} .$$
Find a 4×4 matrix C for which the linear map h given by multiplication by C has the property that both $h(g(x)) = x$ and $g(h(y)) = y$ for all x and all y in $\mathbf{R}^4$.

2. Let f be a linear map from $\mathbf{R}^3$ to $\mathbf{R}^3$ for which
   (a) $f(1, 0, 0) = (1, 2, 3)$.
   (b) $f(0, 1/2, 0) = (3, 2, 1)$.
   (c) $f(-1, 0, 2) = (4, -6, 2)$.

   Find all possible 3×3 matrices A for which the formula $f(x) = Ax$ is valid for all x in $\mathbf{R}^3$.

   Hint: Use the rules for abstract linearity to work out what happens under f to $(0, 1, 0)$ and $(0, 0, 1)$.

3. For a given real number θ find a 2×2 matrix R_θ for which the linear function ρ defined by $\rho(x) = R_\theta x$ is the counterclockwise rotation of the plane through the angle of (radian) measure θ.

   Hint: First work out the four special cases where θ takes the values 0, $\pi/2$, π, and $3\pi/2$.

4. Find a 3×3 matrix S for which the linear function σ given by $\sigma(x) = Sx$ is the reflection of $\mathbf{R}^3$ in the xz plane (where the 2^{nd} coordinate $y = 0$).