Linear Algebra (Math 220)
Assignment due Tuesday, February 26

1 Reading

Relevant Reading

- Lay § 4.2
- Hefferon §§ 3.II – 3.III
- Matthews §§ 3.1 – 3.2

2 Exercises

1. Let C be the 4×4 matrix

$$\begin{pmatrix} 1 & 2 & 0 & 2 \\ -2 & -1 & 3 & 2 \\ -2 & 2 & 6 & -1 \\ 1 & 0 & -2 & 0 \end{pmatrix},$$

   and let f be the linear map (or function) from $\mathbf{R}^4$ to $\mathbf{R}^4$ defined by the formula

$$y \; = \; f(x) \; = \; Cx \quad .$$

   (a) Find all solutions of $f(x) = (0,0,0,0)$.
   (b) Find all solutions of $f(x) = (1,-2,-2,1)$ with $x_3 = 0$.
   (c) Find all solutions of $f(x) = (1,-2,-2,1)$.
   (d) Find all solutions of $f(x) = (-1,-7,2,1)$ with $x_3 = 0$.
   (e) Find all solutions of $f(x) = (-1,-7,2,1)$.
   (f) What is the kernel of f?
   (g) Find equations that characterize the image of f.

2. Let G be the 4×4 matrix

$$\begin{pmatrix} 1 & 2 & 0 & 1 \\ -2 & -1 & 1 & 1 \\ -1 & 4 & 2 & 5 \\ 5 & 7 & -1 & 2 \end{pmatrix},$$

   and let g be the linear map (or function) from $\mathbf{R}^4$ to $\mathbf{R}^4$ defined by the formula

$$y \; = \; g(x) \; = \; Gx \quad .$$

   Solve each of the following systems of 4 linear equations in 4 unknowns x_1, x_2, x_3 and x_4.

   (a) $g(x) = (0,0,0,0)$.
   (b) $g(x) = (1,-1,1,3)$ with $x_3 = 0$.
   (c) $g(x) = (1,-1,1,4)$ with $x_3 = 0$.
   (d) $g(x) = (1,-1,1,4)$ with $x_3 = x_4 = 0$.
   (e) $g(x) = (3,-1,2,1)$ with $x_3 = 0$.
   (f) $g(x) = (3,-1,7,10)$ with $x_3 = 0$.
   (g) What is the kernel of g?
   (h) Find equations that characterize the image of f.

3. Let M be an $m \times n$ matrix, and let $\varphi(x) = Mx$. Let a and b be any two points of $\mathbf{R}^n$. Show that $\varphi(a) = \varphi(b)$ if and only if $a - b$ lies in the kernel of φ.