

# Math 220 Assignment

December 5, 2001

## The Quadratic Form Associated With a Symmetric Matrix

If  $S$  is an  $n \times n$  symmetric matrix and if  $x$  is the column of coordinates of a point  $p$  in an  $n$ -dimensional vector space relative to a given basis, then the matrix product

$$x'Sx$$

is a  $1 \times 1$  matrix whose sole entry  $Q_S(p)$  is a scalar function of the point  $p$  that is a polynomial of degree 2 in the coordinates  $x_1, x_2, \dots, x_n$  of  $p$ . The function  $Q_S$  is called a quadratic form, and  $S$  is the matrix of the quadratic form relative to the given coordinate system.

## Due Friday, December 7

1. If with a change of basis each point  $p$  that is represented in a given basis by  $x$  is represented relative to another basis by  $y$  where  $x = Ay$  for a given invertible matrix  $A$ , what is the matrix relative to the second basis of the quadratic form that has matrix  $S$  relative to the given basis?
2. Let  $f(x_1, x_2) = 2x_1x_2$ .
  - (a) For what  $2 \times 2$  symmetric matrix  $S$  is  $Q_S = f$ ?
  - (b) Find a basis of  $\mathbf{R}^2$  consisting of mutually perpendicular unit vectors relative to which the matrix of  $f$  is a diagonal matrix.
3. Let  $S$  be the  $3 \times 3$  symmetric matrix

$$\begin{pmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{pmatrix} .$$

- (a) Find a diagonal matrix that represents  $Q_S$  relative to some basis of  $\mathbf{R}^3$  consisting of mutually perpendicular unit vectors.
- (b) What is the largest value achieved by  $Q_S$  on the unit sphere  $x_1^2 + x_2^2 + x_3^2 = 1$ ?

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