Holiday Puzzle Solution - Groundhog Day, 2009

THIS YEAR: A SPHERE PACKING PROBLEM

Suppose you have a cube where each side has length 1. You wish to put spheres (whose radii may differ) into the cube so the sum of the volumes of the spheres is at least 2/3. The spheres may be tangent to each other but may not otherwise overlap. The spheres may be tangent to the cube but otherwise must be inside the cube.

Describe how to do this sphere packing. Try to use close to as few spheres as possible.

Solution: Here is one possibility for the spheres. While I have tried to use relatively few spheres, other approaches perhaps might give fewer spheres.

Start with one sphere with center $(1/2, 1/2, 1/2)$ and radius $1/2$. This sphere has volume $(4/3)\pi(1/2)^3 = \pi/6$. This value is approximately 0.523598.

We shall put 8 spheres near each corner as follows. The centers are at (a, b, c) where $a, b, c, \in \{A, 1-A\}$ with $A = 1-(\sqrt{3}/2)$. Each sphere has radius A. Note that these spheres do not overlap, lie within the cube, and do not overlap the first sphere except at a tangent point. The total volume of these spheres is approximately 0.080583.

Now pick 24 spheres. The centers are of the forms (a, b, c), (a, c, b), and (c, a, b) where $a, b \in \{B, 1-B\}$ and $c \in \{0.36, 0.64\}$ such that $B = 0.0927$. Each sphere has radius B. Note that these spheres all lie in the cube and do not overlap each other. Furthermore, these spheres do not overlap the 9 spheres previously described. These 24 spheres have total volume approximately 0.080082; any 19 of them have total volume approximately 0.063398. Thus we have 28 non-overlapping spheres in this cube so that the spheres have total volume approximately 0.667579, which is greater than 2/3.

No one submitted a valid description of the spheres, although one incorrect solution (with one sphere albeit too large to fit in the cube) was received.

I do plan to have another Holiday Puzzle next holiday season. Look for it on the Web at `http://math.albany.edu/~martinhi/puzzle.html`

M. Hildebrand
University at Albany
Department of Mathematics and Statistics
Albany, NY 12222
E-mail: `martinhi@math.albany.edu`