# Math 520B Written Assignment No. 4

## due November 22, 2004

**Directions.** It is intended that you work these as exercises. Although you may refer to books for definitions and standard theorems, searching for solutions to these written exercises either in books or in online references should not be required and is undesirable. If you make use of a reference other than class notes, you must properly cite that use.

For some of these exercises the use of a computer algebra system is suggested.

You may not seek help from others.

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1. Let $f$ be the $\mathbf{Z}$-linear map from $\mathbf{Z}^4$ to $\mathbf{Z}^3$ defined by $f(x) = Mx$ for all $x \in \mathbf{Z}^4$ where M is the $3 \times 4$ matrix
$$M = \begin{pmatrix} 24 & 30 & -36 & -42 \\ 14 & 24 & -34 & -44 \\ 34 & 48 & -62 & -76 \end{pmatrix} .$$
Find cyclic groups whose product is isomorphic to the cokernel of $f$.

2. Find one matrix in each of the similarity classes of matrices over the field $\mathbf{C}$ (of complex numbers) that share the characteristic polynomial $t^3 - t^2 - t + 1$.

3. Let $A$ be the $5 \times 5$ matrix in the field $\mathbf{F}_2 = \mathbf{Z}/2\mathbf{Z}$
$$A = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix} .$$
Find a direct sum of companion matrices that is similar to $A$.

4. Let $M$ be the $6 \times 6$ rational matrix
$$M = \begin{pmatrix} 102 & -52 & 71 & -21 & 16 & -16 \\ 324 & -157 & 228 & -69 & 78 & -57 \\ 357 & -182 & 244 & -66 & 49 & -60 \\ 532 & -286 & 356 & -88 & 19 & -82 \\ 36 & -26 & 20 & 3 & -24 & -6 \\ 532 & -286 & 356 & -90 & 19 & -80 \end{pmatrix} .$$
Find the sequence of successively divisible invariant factors as well as the minimal and characteristic polynomials of $M$.

5. Prove that in a principal ideal domain every non-zero prime ideal is a maximal (proper) ideal.