Math 520B Written Assignment No. 3

due Monday, October 18, 2004

Directions. It is intended that you work these as exercises. Although you may refer to books for definitions and standard theorems, searching for solutions to these written exercises either in books or in online references should not be required and is undesirable. If you make use of a reference other than class notes, you must properly cite that use.

You may not seek help from others.

---

1. Let F be a field. Find a familiar ring isomorphic to the tensor product of matrix algebras

$$\mathrm{M}_p(F) \otimes_F \mathrm{M}_q(F) \quad .$$

2. Let R and S be commutative rings with S an R-algebra. Show that the polynomial rings over R and S are related by a canonical isomorphism

$$S[t] \cong R[t] \otimes_R S \quad .$$

3. Let R be a ring, and let M and N be R-bi-modules. Then both $M \otimes_R N$ and $N \otimes_R M$ are R-bi-modules. Under what circumstances are $M \otimes_R N$ and $N \otimes_R M$ isomorphic R-bi-modules?

4. A (left) module M on a commutative ring R always gives rise to a bi-R-module with the property that for all $r \in R$ and $m \in M$ one has $r \cdot m = m \cdot r$. Let R be the polynomial ring $\mathbf{F}_2[t]$ where $\mathbf{F}_2$ is the field of two elements, and let M be the two-dimensional Cartesian space $\mathbf{F}_2 \times \mathbf{F}_2$. Give an example of an R-bi-module structure on M for which the right multiplication by a scalar is not equal to the left multiplication.

5. Let K be a field, L a (finite) Galois extension of K, G the Galois group, and $R = KG$ the group ring of G with coefficients in K. Observe that L and R have the same dimension as vector spaces over K.

   (a) Show that there is an obvious R-module structure on L.

   (b) Formulate in terms of the study of L as an extension of K without reference to the notion of group ring what it means for L to be isomorphic as an R-module to R.

Document network location for HTML:

`http://math.albany.edu:8000/math/pers/hammond/course/mat520bf2004/assgt/adc041018.html`