Math 520B Written Assignment No. 1

due Wednesday, September 15, 2004

Directions. It is intended that you work these as exercises. Although you may refer to books for definitions and standard theorems, searching for solutions to these written exercises either in books or in online references should not be required and is undesirable. If you make use of a reference other than class notes, you must properly cite that use.

You may not seek help from others.

If R is a ring, then R^* denotes its multiplicative group and $\mathrm{M}_n(R)$ denotes the ring (relative to coordinate-wise addition and matrix multiplication) of $n \times n$ matrices over R.

If A is a matrix over a ring R, the symbol A_{ij} denotes the element of R that is located in A at row i, column j.

1. Recall that if A is an abelian group, written additively, the set $\mathrm{End}(A)$ of endomorphisms of A, i.e., group homomorphisms from A to itself, is a ring R with addition defined element-wise and with multiplication given by composition of endomorphisms.
   (a) Verify the left and right distributive laws for R.
   (b) Find an explicit computation of $\mathrm{End}(\mathbf{Z} \times \mathbf{Z})$.
2. Prove that for any field F the ring $R = \mathrm{M}_2(F)$ has no non-trivial proper two-sided ideal.
3. If F is a field let $\mathrm{T}_n(F)$ denote the set of *upper triangular* $n \times n$ matrices in F, i.e., the set of all $n \times n$ matrices A over F for which $A_{ij} = 0$ when $i > j$. The set $\mathrm{T}_n(F)$ is a subring R of the full matrix ring $\mathrm{M}_n(F)$. In R the set I of all *strictly upper triangular* matrices, i.e., A with $A_{ij} = 0$ when $i \geq j$, is a 2-sided ideal. Show that the quotient ring R/I is commutative.
4. Find all isomorphism classes of rings with 4 elements.
5. Prove that for any commutative ring R one has

$$\mathrm{M}_n(R)^* = \{A \in \mathrm{M}_n(R) \mid \det(A) \in R^*\} \quad .$$

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