

Math 520A Written Assignment No. 2

due Monday, March 12, 2007

Directions. This assignment should be typeset. You must explain the reasoning underlying your answers. If you make use of a reference other than class notes, you must properly cite its use.

You may not seek help from others on this assignment.

1. If G is any group, H a subgroup, and x, y elements of G , prove that $xH = Hy$ if and only if x and y both belong to the normalizer $N_G(H)$ and determine the same element of $N_G(H)/H$.
2. For each relevant prime p determine the number of Sylow p -subgroups of the symmetric group S_5 .
3. Show that the group $SL_2(\mathbf{F}_3)$ has a normal subgroup of order 8. List the 8 elements of this subgroup, and explain why this group of order 8 cannot be isomorphic to the dihedral group D_4 .
4. Let G be a finite group and H a subgroup of index 3 in G that is not a normal subgroup of G . Show that H contains a subgroup N that is normal in G for which $G/N \cong S_3$.
5. Find an explicit list of groups that represent all isomorphism classes of groups of order 66.