Math 520A Written Assignment No. 3

due Friday, March 18, 2005

Directions. It is intended that you work these as exercises. Although you may refer to books for definitions and standard theorems, searching for solutions to these written exercises either in books or in online references should not be required and is undesirable. If you make use of a reference other than class notes, you must properly cite that use.

You may not seek help from others.

Notation: The following notations will be used.

$\mathbf{Z}/m\mathbf{Z}$	the ring of integers mod m or its additive group
D_m	the semi-direct product $\mathbf{Z}/m\mathbf{Z} \rtimes \mathbf{Z}/2\mathbf{Z}$ for the action of the latter on the former by negation
Q_8	the multiplicative group of integer quaternions of norm 1
$A[t]$	the ring of polynomials with coefficients in A (A commutative ring)
$A[t]/(f(t))A[t]$	the quotient (ring) of $A[t]$ by the ideal of all multiples of $f(t)$

Problems

1. Show that every non-abelian group of order 8 is isomorphic either to D_4 or to Q_8.
   Hint: First show that a non-abelian group of order 8 must have at least one element of order 4.
2. Determine all isomorphism classes of rings of order 4.
3. How does the isomorphism class of the group of automorphisms of Q_8 compare with that of other groups that have arisen in this course?
4. Determine the group of real-linear ring automorphisms of the ring
$$\mathbf{R}[t]/(t^4+1)\mathbf{R}[t] \quad .$$
5. Let G be a finite group and H a subgroup of G having index p in G where p is the smallest prime dividing the order of G. Prove that H must be a normal subgroup of G.