

Math 502

Written Assignment No. 3

due Thursday, April 3, 2008

Directions: Use *Maple* for assistance in responding to the following problems. Please typeset your solutions. Explain what you have done. *Maple* session details are not necessary unless you think it important to include them. Accuracy is important.

Although you may refer to books and notes, you may **not** seek help from others on this written assignment.

Answer the following questions:

1. Determine polynomials $f_0(x), f_1(x), f_2(x), \dots$ such that

$$(xy^3 + 1 + x + x^2y)^3 = \sum_{k \geq 0} f_k(x)y^k .$$

2. How can the length 15 string represented by the vector of ASCII codes

[97, 39, 98, 96, 99, 34, 100, 10, 101, 39, 102, 34, 103, 96, 104]

— which contains a newline and various quoting characters — be entered as an input string in Maple without reference to the ASCII codes?

3. In this problem let p denote the prime 40487.

- (a) Find all non-negative residues $k \pmod{p-1}$ for which

(i) $2^k \equiv 5 \pmod{p}$.

(ii) $5^k \equiv 2 \pmod{p}$.

- (b) Find the smallest positive primitive root modulo the prime p .

- (c) Find the smallest positive primitive root modulo p^2 .

- (d) Find the least non-negative integer k for which

$$10^k \equiv 5 \pmod{p^2} .$$

4. The vector

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[1374123367812492240294819411451017240070232394572901767016863846,
839316502484928463600482820810738376821725250432320253174346182,
1239738327596678208340985633338387227858526317346603640791773335,
1786140405373535737086895440853230782772901282581414488521399167,
500665432788843543145295210921142588622600876314918005209142550,
1134791724566968592421020750873161158581657388017051854867699847,
2289042576082270612059353048240349198604028364564225150215042759]
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represents a sequence of ASCII codes (range 0 to 127) that has been squeezed into blocks of 30 and then encrypted by taking a certain power modulo

$$m = 2468256835981809063232453777824906406995466865716803865301401733 .$$

It may be decrypted by using the exponent 5^{15} .

- (a) What string was encrypted?

- (b) Can you find the encryption exponent that is paired with this decryption exponent?

5. Provide a listing of code for a Maple procedure that takes an **expression** e in two variables x, y as first argument and the variable names as second and third arguments and returns the anonymous **function** of two arguments

$$(x, y) \mapsto x \frac{\partial e}{\partial x} + y \frac{\partial e}{\partial y} .$$

Make sure that the code you submit works correctly.