

Math 502

Written Assignment No. 1

due Tuesday, February 12, 2008

Directions: Use *Maple* for assistance in responding to the following problems. Please typeset your solutions. Explain what you have done. *Maple* session details are not necessary unless you think it important to include them. Accuracy is important.

Although you may refer to books and notes, you may not seek help from others on this written assignment.

Technical comment for this assignment:

- Feel free to make use of code found at <http://www.math.albany.edu/~hammond/maple/>.
- The behavior of *Maple*'s `convert[confrac]` appears to have changed between *Maple* 9 and *Maple* 10.

Let a and b be the integers

$$a = 30246273033735921 \quad b = 1363209853178907 .$$

Answer the following questions:

1. Compute the quotient a/b with 50 digits of precision.
2. What is the remainder upon long division of a by b ?
3. What is the greatest common divisor of a and b ?
4. What is the prime factorization of b ?
5. What number of iterations of the Syracuse function beginning with a is required to reach 1?
6. What is the largest iterate of the Syracuse function beginning with a ?
7. What is the length of the continued fraction expansion of a/b .
8. What cycles do you find when iterating the function “one plus the sum of the squares of the base 15 digits” beginning with integers up to 1000.
9. Evaluate the continued fraction given by the vector $[2,1,2,1,1,4,1,1,6,1,1,8,1,1,10,1,1,12]$ both as the ratio of two integers and as a decimal with 20 digits of precision.
10. What is the length of the repeating pattern in the decimal expansion of $208341/66317$? (This is the 7-th convergent in the continued fraction expansion of π .)

Hint: As far as I know, there is no native *Maple* function dedicated to this task. Because the length of the repeating pattern is rather large, actual computation of the digits in the pattern may be rather time consuming. On the other hand, a close examination of the mathematics involved will disclose that one does not need to compute the digits in order to know the length of the cycle.