# Math 502

# Written Assignment No. 3

## due Wednesday, March 28, 2007

**Directions:** Use *Maple* for assistance in responding to the following problems. Please typeset your solutions. Explain what you have done. *Maple* session details are not necessary unless you think it important to include them. Accuracy is important.

Although you may refer to books and notes, you may **not** seek help from others on this written assignment.

Answer the following questions:

1. Determine polynomials $f_0(x), f_1(x), f_2(x), \ldots$ such that

$$(xy^3 + 1 + x + x^2 y)^3 \quad = \quad \sum_{k \geq 0} f_k(x) y^k \quad .$$

2. How can the length 15 string represented by the vector of ASCII codes

   [97, 39, 98, 96, 99, 34, 100, 10, 101, 39, 102, 34, 103, 96, 104]

   — which contains a newline and various quoting characters — be entered as an input string in Maple without reference to the ASCII codes?

3. In this problem let $p$ denote the prime 40487.

   (a) Find all non-negative residues $x \pmod p$ for which $2^x \equiv 5 \pmod p$.
   (b) Find the smallest (positive) primitive root modulo the prime $p$.
   (c) Find the smallest (positive) primitive root modulo $p^2$.
   (d) Find the least non-negative integer $x$ for which

   $$10^x \equiv 5 \pmod{p^2} \quad .$$

4. The vector

   [1374123367812492240294819411451017240070232394572901767016863846,
   8393165024849284636004828208107383768217252504323202531743461 82,
   1239738327596678208340985633338872278585263173466036407917733 35,
   1786140405373535737086895440853230782772901282581414488521399167,
   5006654327888435431452952109211425886226008763149180052091425 50,
   1134791724566968592421020750873161158581657388017051854867699847,
   2289042576082270612059353048240349198604028364564225150215042759]

   represents a sequence of ASCII codes (range 0 to 127) that has been squeezed into blocks of 30 and then encrypted by taking a certain power modulo

   $$m \quad = \quad 2468256835981809063232453777824906406995466865716803865301401733 \quad .$$

   It may be decrypted by using the exponent $5^{15}$.

   (a) What string was encrypted?
   (b) Can you find the encryption exponent that is paired with this decryption exponent?

5. Provide a listing of code for a Maple procedure that takes an **expression** $e$ in two variables $x, y$ as first argument and the variable names as second and third arguments and returns the anonymous **function** of two arguments

   $$(x, y) \longmapsto x\frac{\partial e}{\partial x} + y\frac{\partial e}{\partial y} \quad .$$

   Make sure that the code you submit works correctly.