

# Advanced Linear Algebra (Math 424/524)

## Review Exercises

September 18, 2002

1. Find the reduced row echelon form of the matrix

$$\begin{pmatrix} 0 & 0 & 2 & 0 \\ -1 & 4 & 0 & 2 \\ 3 & -2 & 0 & 1 \\ 0 & 1 & -1 & 0 \end{pmatrix} .$$

2. Find the determinant of the  $3 \times 3$  matrix

$$\begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix} .$$

3. Find the inverse of the orthogonal matrix

$$\frac{1}{7} \begin{pmatrix} 2 & 3 & 6 \\ 6 & 2 & -3 \\ 3 & -6 & 2 \end{pmatrix} .$$

4. Let  $T$  be the linear transformation from  $\mathbf{R}^3$  to  $\mathbf{R}^2$  given by

$$T(x_1, x_2, x_3) = (3x_2 - x_3, x_1 + 4x_2 + x_3) .$$

Find the unique  $2 \times 3$  matrix  $A$  such that

$$T(x) = Ax$$

for each  $x$  in  $\mathbf{R}^3$ .

5. Find a basis for the vector subspace of  $\mathbf{R}^4$  that consists of all solutions of the system of linear equations

$$\begin{aligned} x_1 - 2x_3 + x_4 &= 0 \\ x_2 + 3x_3 - 2x_4 &= 0 \end{aligned}$$

6. Let  $f$  be the linear function from  $\mathbf{R}^4$  to  $\mathbf{R}^4$  that is defined by  $f(x) = Mx$  where  $M$  is the matrix

$$\begin{pmatrix} 1 & -1 & -2 & 0 \\ -1 & 2 & 0 & -3 \\ 2 & 0 & -1 & 1 \\ 0 & -1 & 2 & 3 \end{pmatrix} .$$

(a) Find a basis of the kernel of  $f$ .

(b) Find one or more non-redundant linear equations that characterize the image of  $f$ , i.e., equations for which the set of common solutions is the image of  $f$ .

7. Give an explicit description of the set of all  $n \times n$  matrices that are similar to the  $n \times n$  identity matrix.

8. Let  $g$  be the linear function from  $\mathbf{R}^3$  to  $\mathbf{R}^3$  that is defined by  $g(x) = Rx$  where  $R$  is the matrix

$$\begin{pmatrix} 2 & 1 & 2 \\ 2 & -2 & -1 \\ 1 & 2 & -2 \end{pmatrix} .$$

Find as many as possible non-parallel eigenvectors of  $g$ , i.e., non-zero vectors  $x$  in  $\mathbf{R}^3$  for which  $g(x)$  is a scalar multiple of  $x$ .