Advanced Linear Algebra (Math 424/524)

8 September 2002

These are just exercises to aid in reviewing your previous knowledge of linear algebra.

1. Find a parametric description of the set of all solutions (x, y, z) of the system of linear equations

$$\begin{array}{rcl} x - 2y + z & = & 0 \\ 2x - 3y - z & = & 6 \end{array} \quad .$$

2. Find the inverse of the matrix

$$\begin{pmatrix} 1 & -2 \\ 1 & -3 \end{pmatrix} \quad .$$

3. Let M be the 3×3 matrix that is given by

$$M \quad = \quad \begin{pmatrix} 0 & 2 & 4 \\ 1 & 0 & 1 \\ 3 & 1 & 0 \end{pmatrix} \quad .$$

   Find the determinant of M.

4. Let

$$f : \mathbf{R}^3 \longrightarrow \mathbf{R}^3$$

   be the map defined by $f(x) = Mx$, where M is the matrix

$$M \quad = \quad \begin{pmatrix} 0 & 2 & 4 \\ 1 & 0 & 1 \\ -2 & 1 & 0 \end{pmatrix} \quad .$$

   (a) What is the rank of the matrix M?

   (b) Find an equation for the image of f.

   (c) Find a parametric representation of the fiber of f over the point $(6, 2, -1)$.

   (d) Find a point p in the image of f such that the vector drawn from the origin to p is perpendicular to the vector drawn from the origin to $(6, 2, -1)$.