Written Assignment No. 4

due Monday, November 21, 2005

General Directions: Written assignments should be submitted typeset. What you submit must represent your own work.

Assigned Exercises

1. Let A be a domain. Prove that the group of units in the polynomial ring $A[x]$ is isomorphic to the group of units in A.

2. Let F be a finite field with $|F| = q$. Let R be the set of all functions $F \longrightarrow F$. Observe that the number $|R|$ of such functions is q^q. Observe that R is an abelian group under pointwise addition of functions, i.e.,

$$(f + g)(x) \; = \; f(x) + g(x) \; ,$$

and that R becomes a ring when multiplication is defined pointwise, i.e.,

$$(f \cdot g)(x) \; = \; f(x)\, g(x) \quad .$$

   (a) What ring homomorphism $\phi : F[x] \to R$ has the properties that (i) ϕ applied to a polynomial of degree 0 is the corresponding constant function and (ii) ϕ applied to the polynomial x is the identity function?

   (b) Find a polynomial of degree q in the kernel of ϕ.

   (c) What is the kernel of ϕ?

   (d) Show that ϕ is surjective.