Classical Algebra

Written Assignment No. 2

due Tuesday, October 7, 2008

Directions

Written assignments must be typeset.

While it is neither necessary nor desirable to show small details of computation, you must indicate what you are doing, give major steps in computation, and explain any reasoning used.

Accuracy is important. With 5 problems in an assignment worth 10 points, there is limited room for partial credit on a problem.

Problems

1. Find (i) the least non-negative residue and (ii) the residue of least absolute value for 2^{2123} modulo 1025.

2. Find all points (x, y) with integer coordinates on the line

$$301x + 259y \;=\; 35 \quad .$$

3. List *all* solutions that are distinct mod 50 for each of the following congruences:

   (a) $3x \equiv 1 \pmod{50}$.

   (b) $3x \equiv 16 \pmod{50}$.

   (c) $28x \equiv 41 \pmod{50}$.

   (d) $47x \equiv 21 \pmod{50}$.

   (e) $40x \equiv 55 \pmod{50}$.

4. List the *number* of distinct solutions mod 445357 for each of the following congruences:

   (a) $66x \equiv 1133 \pmod{445357}$

   (b) $66x \equiv 256 \pmod{445357}$

   (c) $66x \equiv 33 \pmod{445357}$

5. Prove that a and b have least common multiple ab if there exist integers r and s such that

$$ar + bs \;=\; 1 \quad .$$