

Classical Algebra

Written Assignment No. 2

due Thursday, October 12, 2006

(corrected Oct 5 2006 12:10 p.m.)

Directions

Written assignments must be typeset.

While it is neither necessary nor desirable to show small details of computation, you must indicate what you are doing, give major steps in computation, and explain any reasoning used.

Accuracy is important. With 5 problems in an assignment worth 5 points, there will be no room for partial credit on a problem.

If you are in the writing intensive division of the course, you must complete each written assignment in a satisfactory way. This may require re-submission, possibly more than once, after the initial evaluation.

Problems

1. Find (i) the least non-negative residue and (ii) the residue of least absolute value for 2^{213} modulo 1025.

2. Find all points (x, y) with integer coordinates on the line

$$129x + 111y = 33 \quad .$$

3. List *all* solutions that are distinct mod 40 for each of the following congruences:

(a) $3x \equiv 1 \pmod{40}$.

(b) $3x \equiv 16 \pmod{40}$.

(c) $28x \equiv 41 \pmod{40}$.

(d) $47x \equiv 21 \pmod{40}$.

(e) $25x \equiv 55 \pmod{40}$.

4. List the *number* of distinct solutions mod 283409 for each of the following congruences:

(a) $42x \equiv 791 \pmod{283409}$

(b) $42x \equiv 256 \pmod{283409}$

(c) $42x \equiv 14 \pmod{283409}$

5. Prove that a and b have least common multiple ab if there exist integers c and d such that

$$ac + bd = 1 \quad .$$