Math 220 Class Slides

http://math.albany.edu/pers/hammond/course/mat220/

February 12, 2008

1 Assignment due February 14

Expect a **quiz**.
Read Matthews, §§ 8.1 – 8.4
Exercises:
Matthews, **185:** 1 – 7

2 Inverting a Matrix with Row Operations

1. If M has size $n \times n$, form the hyper-augmented matrix (MI_n) of size $n \times 2n$, where I_n is the $n \times n$ identity matrix.
2. Perform row operations on (MI_n) to bring its first n columns into reduced row echelon form.
3. **IF** the first n columns of the new matrix form I_n, then
   (a) M is invertible.
   (b) the last n columns of the new matrix form the inverse matrix Q.

3 Feb. 12 Assignment, No. 2

To find the inverse of the matrix

$$\begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix}$$

Form the 2×4 matrix

$$\begin{pmatrix} 1 & -1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

Then perform row operations

$$\begin{pmatrix} 1 & -1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

$$(R_1 \leftrightarrow R_2) \begin{pmatrix} 1 & 0 & 0 & 1 \\ 1 & -1 & 1 & 0 \end{pmatrix}$$

$$(R_2 \to R_2 - R_1) \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & -1 \end{pmatrix}$$

$$(R_2 \to - R_2) \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 1 \end{pmatrix}$$

Answer:

$$\begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix}$$

4 Feb. 7 Assignment, No. 1

$$\mathbf{R}^4 \xrightarrow{f} \mathbf{R}^3 \qquad f(X) \;=\; f_A(X) \;=\; AX$$

where

$$A \;=\; \begin{pmatrix} 2 & 3 & 1 & -4 \\ 3 & -2 & -1 & 5 \\ 5 & 1 & 0 & 1 \end{pmatrix}$$

- To handle all three tasks perform row operations on the augmented matrix

$$\begin{pmatrix} 2 & 3 & 1 & -4 & y_1 \\ 3 & -2 & -1 & 5 & y_2 \\ 5 & 1 & 0 & 1 & y_3 \end{pmatrix}$$

- The reduced row echelon form:

$$\begin{pmatrix} 1 & 0 & -1/13 & 7/13 & (2y_1+3y_2)/13 \\ 0 & 1 & 5/13 & -22/13 & (3y_1-2y_2)/13 \\ 0 & 0 & 0 & 0 & y_3-y_1-y_2 \end{pmatrix}$$

- There are no solutions unless $y_3 \;=\; y_1 + y_2$.
- When $y_3 \;=\; y_1 + y_2$, the transformed system of linear equations is:

$$\begin{aligned} x_1 - \frac{1}{13}x_3 + \frac{7}{13}x_4 &= \frac{2y_1+3y_2}{13} \\ x_2 + \frac{5}{13}x_3 - \frac{22}{13}x_4 &= \frac{3y_1-2y_2}{13} \end{aligned}$$

- x_1 and x_2 may be expressed as functions of x_3 and x_4.
- The variables corresponding to pivot columns may be expressed as functions of the other variables.

- Every solution for the case when $Y = 0$ has the form

$$Z = s \begin{pmatrix} 1/13 \\ -5/13 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} -7/13 \\ 22/13 \\ 0 \\ 1 \end{pmatrix}$$

as the parameters s and t range over all real values — a "plane" in the 4-dimensional space $\mathbf{R}^4$.

- The set of Y for which $f(X) = Y$ has at least one solution — the image of f — is the plane in $\mathbf{R}^3$ given by the equation $y_1 + y_2 - y_3 = 0$.

- Every solution W of

$$AX = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}$$

has the form

$$W = Z + \begin{pmatrix} 5/13 \\ 14/13 \\ 0 \\ 0 \end{pmatrix}$$

where Z is given by the formula above.

5 Feb. 12 Assignment, No. 1

Let R(s, t) be the function from $\mathbf{R}^2$ to $\mathbf{R}^3$ defined by

$$R(s,t) = (s + 2t, -2s - t, -2s + 2t) \quad .$$

1. Find equation(s) that characterize the set S of all points (x, y, z) in $\mathbf{R}^3$ that arise as $R(s, t)$ for at least one pair (s, t).
2. What kind of subset of $\mathbf{R}^3$ is S?

6 Assgt., No. 1: Generic Augmented Matrix

$$\begin{pmatrix} 1 & 2 & x \\ -2 & -1 & y \\ -2 & 2 & z \end{pmatrix}$$

Technique:

1. Perform row operations
2. Bring the coefficient portion to RREF
3. Form the resulting system
4. Apply common sense

7 Assgt., No. 1: Row Operations

$$\begin{pmatrix} 1 & 2 & x \\ -2 & -1 & y \\ -2 & 2 & z \end{pmatrix}$$

$$(R_2 \to R_2 + 2R_1) \begin{pmatrix} 1 & 2 & x \\ 0 & 3 & 2x+y \\ -2 & 2 & z \end{pmatrix}$$

$$(R_3 \to R_3 + 2R_1) \begin{pmatrix} 1 & 2 & x \\ 0 & 3 & 2x+y \\ 0 & 6 & 2x+z \end{pmatrix}$$

$$(R_3 \to R_3 - 2R_2) \begin{pmatrix} 1 & 2 & x \\ 0 & 3 & 2x+y \\ 0 & 0 & -2x-2y+z \end{pmatrix}$$

(Now in row echelon form, but not *reduced* row echelon form)

$$\left(R_2 \to \frac{1}{3}R_2\right) \begin{pmatrix} 1 & 2 & x \\ 0 & 1 & (2x+y)/3 \\ 0 & 0 & z-2x-2y \end{pmatrix}$$

$$(R_1 \to R_1 - 2R_2) \begin{pmatrix} 1 & 0 & -(x+2y)/3 \\ 0 & 1 & (2x+y)/3 \\ 0 & 0 & z-2x-2y \end{pmatrix}$$

8 Assgt., No 1: Application

Remember that s and t are the unknowns, while x, y, and z are "on the right".
The resulting linear system:

$$\begin{aligned} s &= -\frac{x+2y}{3} \\ t &= \frac{2x+y}{3} \\ 0 &= z-2x-2y \end{aligned}$$

- s and t can be expressed in terms of x, y, and z
- **No solution** unless $z = 2x+2y$
- S is the plane in $\mathbf{R}^3$ of all points satisfying the equation $z = 2x+2y$