Math 220 Class Slides

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January 29, 2008

1 Linear Equations and Matrices

- A system of linear equations is represented by its augmented matrix.
- High school manipulations of the equations correspond to row operations on the augmented matrix.
- Maneuvers to put the matrix in the form where the system is (essentially) solved involve proceeding in a systematic way.

2 Vector Arithmetic

- Vector addition

$$(x_1, x_2, \ldots, x_n) + (y_1, y_2, \ldots, y_n) \;=\; (x_1 + y_1, x_2 + y_2, \ldots, x_n + y_n)$$

- Multiplication of a vector by a scalar

$$c(x_1, x_2, \ldots, x_n) \;=\; (cx_1, cx_2, \ldots, cx_n)$$

3 Elementary Row Operations

There are three.

1. Replace a row by its sum with a multiple of another row.
2. Switch two rows.
3. Replace a row by a **non-zero** multiple of itself.

4 Row Echelon Form

A matrix is in *row echelon form* if

- All non-zero rows precede all zero rows.
- The leading non-zero elements in the non-zero rows are staggered.

5 Reduced Row Echelon Form

A matrix is in *reduced row echelon form* if

- It is in row echelon form.
- The first non-zero element in a non-zero row is a 1.
- A leading 1 is the only non-zero element in its column.

6 Example

$$\begin{aligned} x_1 - 2x_2 + x_3 &= 0 \\ 2x_2 - 8x_3 &= 8 \\ -4x_1 + 5x_2 + 9x_3 &= -9 \end{aligned}$$

Augmented matrix:

$$\begin{pmatrix} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ -4 & 5 & 9 & -9 \end{pmatrix}$$

7 Example Solved

$$\begin{pmatrix} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ -4 & 5 & 9 & -9 \end{pmatrix}$$

$$R_3 \to R_3 + 4R_1 \qquad \begin{pmatrix} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 0 & -3 & 13 & -9 \end{pmatrix}$$

$$R_2 \to \frac{1}{2}R_2 \qquad \begin{pmatrix} 1 & -2 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & -3 & 13 & -9 \end{pmatrix}$$

$$R_3 \to R_3 + 3R_2 \qquad \begin{pmatrix} 1 & -2 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & 0 & 1 & 3 \end{pmatrix}$$

$$R_2 \to R_2 + 4R_3 \qquad \begin{pmatrix} 1 & -2 & 1 & 0 \\ 0 & 1 & 0 & 16 \\ 0 & 0 & 1 & 3 \end{pmatrix}$$

$$R_1 \to R_1 + 2R_2 \qquad \begin{pmatrix} 1 & 0 & 1 & 32 \\ 0 & 1 & 0 & 16 \\ 0 & 0 & 1 & 3 \end{pmatrix}$$

$$R_1 \to R_1 - R_3 \qquad \begin{pmatrix} 1 & 0 & 0 & 29 \\ 0 & 1 & 0 & 16 \\ 0 & 0 & 1 & 3 \end{pmatrix}$$

$$\begin{aligned} x &= 29 \\ y &= 16 \\ z &= 3 \end{aligned}$$