Linear Algebra (Math 220)
Assignment due Tuesday, April 22

1 Preparation

Expect a **quiz**.

Relevant Reading:

Lay §§ 6.3 – 6.4
Hefferon § 3.VI

2 Exercises

1. Let U denote the 3×3 matrix

$$\frac{1}{3}\begin{pmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ -1 & 2 & -2 \end{pmatrix} ,$$

   and let φ be the linear function from $\mathbf{R}^3$ to $\mathbf{R}^3$ defined by $\varphi(x) = Ux$ for all x in $\mathbf{R}^3$.

   (a) Show that the columns of U are mutually perpendicular vectors in $\mathbf{R}^3$ of length 1.

   (b) Show that the rows of the transposed matrix ${}^tU$ are mutually perpendicular vectors in $\mathbf{R}^3$ of length 1.

   (c) Compute the matrix product ${}^tUU$.

   (d) Show that φ is an invertible linear function, and find the matrix for φ^{-1}.

   (e) Explain why the function φ preserves lengths and angles. *Hint.* What effect does applying φ have on the "dot product" of two vectors?

2. Let P_2 be the vector space of polynomials of degree at most 2. Define a scalar product (analogous to "dot" product) Γ on P_2 with the formula

$$\Gamma(f, g) \ = \ \int_0^1 f(t)g(t)dt \quad .$$

   Find the orthogonal complement, relative to Γ, of the subspace consisting of the constant polynomials.

3. Find the matrix, relative to the standard basis of $\mathbf{R}^3$, of the linear map from $\mathbf{R}^3$ to $\mathbf{R}^3$ that for each x in $\mathbf{R}^3$ sends x to its orthogonal projection on the plane in $\mathbf{R}^3$ defined by the linear equation

$$2x - y + 2z \ = \ 0 \quad .$$