Linear Algebra (Math 220)
Assignment due Thursday, April 10

1 Preparation

Expect a quiz.

Relevant Reading:

Lay §§ 2.2, 3.2, 3.3
Hefferon § 3.IV

Bear in mind the following:

Definition. A square matrix is called an *elementary matrix* (or *elementary reduction matrix*) if it is the matrix that results from applying a single elementary row operation to an identity matrix of the same size.

2 Exercises

1. Describe the elementary row operations corresponding to each of the following elementary matrices:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} .$$

2. Find the determinants of each of the elementary matrices in exercise 1.

3. Find the inverses of each of the elementary matrices in exercise 1.

4. Find the product of the four elementary matrices in exercise 1.

5. Show that if E is an $m \times m$ elementary matrix and M is any $m \times n$ matrix, then the product matrix EM is the matrix obtained by applying the elementary row operation corresponding to E to the matrix M.

6. Show that if E is an $m \times m$ elementary matrix and M is any $m \times n$ matrix, then

$$\det(EM) \;=\; \det(E)\det(M) \quad .$$