

Linear Algebra (Math 220)

Assignment due Thursday, March 13

Midterm Test: Tuesday, March 18

1 Preparation

Expect a quiz.

Bring Questions in preparation for the Midterm Test.

2 Exercises

1. When M is an $m \times n$ matrix, the phrase “corresponding linear function” will denote the linear function

$$\mathbf{R}^n \xrightarrow{f_M} \mathbf{R}^m$$

defined by

$$f_M(x) = Mx \text{ for } x \text{ in } \mathbf{R}^n .$$

In the case $m = 2$, $n = 3$

$$M = \begin{pmatrix} 3 & 6 & 0 \\ 2 & 4 & 1 \end{pmatrix}$$

compute each of the following items both for (i) M itself and for (ii) its reduced row echelon form:

- The set of linear combinations of the columns.
 - The set of linear combinations of the rows.
 - The set of linear relations among the columns.
 - The set of linear relations among the rows.
 - The kernel of the corresponding linear function.
 - The image of the corresponding linear function.
2. Let Q_3 be the 4-dimensional vector space consisting of all polynomials of degree 3 or less, and let

$$\mathbf{v} = \{1, t, t^2, t^3\}$$

be the familiar basis of Q_3 . Let $Q_3 \xrightarrow{\phi} Q_3$ be the linear map that is defined by

$$\phi(P) = P'' + 3P' + 2P ,$$

where P' and P'' denote the first and second derivatives of P . Find the matrix of ϕ with respect to the basis \mathbf{v} , i.e., find the 4×4 matrix R that appears in the transport diagram

$$\begin{array}{ccc} Q_3 & \xrightarrow{\phi} & Q_3 \\ \alpha_{\mathbf{v}} \uparrow & & \uparrow \alpha_{\mathbf{v}} \\ \mathbf{R}^4 & \xrightarrow{f_M} & \mathbf{R}^4 \end{array} .$$