Linear Algebra (Math 220)

Assignment due Thursday, February 7

1 Reading

Read §§ 2.2 – 2.3 in Matthews.

2 Exercises

1. Let A be the 3×4 matrix

$$A = \begin{pmatrix} 2 & 3 & 1 & -4 \\ 3 & -2 & -1 & 5 \\ 5 & 1 & 0 & 1 \end{pmatrix} .$$

Let f be the function from $\mathbf{R}^4$ to $\mathbf{R}^3$ given by $f(x) = Ax$.

(a) Find all points x in $\mathbf{R}^4$ for which $f(x) = 0$.

(b) Find all points x in $\mathbf{R}^4$ for which

$$f(x) = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} .$$

(c) Characterize the set of points y in $\mathbf{R}^3$ for which the relation $f(x) = y$ holds for at least one point x in $\mathbf{R}^4$.

2. Let M be the matrix

$$\begin{pmatrix} 1 & 5 & -2 \\ -2 & 4 & -3 \\ -1 & -3 & 1 \end{pmatrix} ,$$

and let g be the function from $\mathbf{R}^3$ to $\mathbf{R}^3$ given by $g(x) = Mx$.

(a) Find all points x in $\mathbf{R}^3$ for which $g(x) = 0$.

(b) Find all points x in $\mathbf{R}^3$ for which

$$g(x) = \begin{pmatrix} 1 \\ -5 \\ 3 \end{pmatrix} .$$

(c) Find all points x in $\mathbf{R}^3$ for which

$$g(x) = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} .$$

(d) Characterize the set of points y in $\mathbf{R}^3$ for which the relation $g(x) = y$ holds for at least one point x in $\mathbf{R}^3$.