Math 220 Assignment

November 30, 2001

The Characteristic Equation

If, relative to a given coordinate system in an n-dimensional vector space, the columns of an invertible $n \times n$ matrix A form a basis of that vector space relative to which a linear transformation that is represented in the given coordinate system by a matrix M is diagonalized, i.e., represented by a diagonal matrix D, then

$$A^{-1}MA \;=\; D \quad .$$

Equivalently $MA = AD$, and, taking the j^{th} column one sees that

$$MA_j \;=\; (MA)_j \;=\; (AD)_j \;=\; AD_j \;=\; d_{jj}A_j \quad .$$

Thus, each member A_j of the diagonalizing basis must lie in the kernel of the linear function represented in the given coordinate system by the matrix $M - t1_n$, where 1_n denotes the $n \times n$ identity matrix, when $t = d_{jj}$. Thus, each A_j may be found by finding the kernel of $M - t1_n$ when $t = d_{jj}$, and the diagonal elements d_{jj} of D may be found among the roots of the characteristic polynomial equation

$$\det\left(M - t1_n\right) \;=\; 0 \quad .$$

Due Monday, December 3

1. Find the characteristic polynomial and its roots for each of the matrices

$$\begin{pmatrix} 3 & 4 \\ 4 & -3 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} -1 & 0 \\ 1 & 1 \end{pmatrix} \quad .$$

2. Let S be the 3×3 matrix

$$\begin{pmatrix} 10 & -6 & -2 \\ -6 & 5 & -8 \\ -2 & -8 & 3 \end{pmatrix} \quad .$$

   Find an orthogonal matrix U and a diagonal matrix D such that

$$S \;=\; UDU^{-1} \quad .$$

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