Math 220 Assignment

October 26, 2001

Due Monday, October 29

The set of polynomials

$$f(X) \;=\; \sum_{j=0}^{d} c_j X^j \;=\; c_0 + c_1 X + c_2 X^2 + \ldots + c_d X^d$$

of degree d (or less if c_d happens to be 0) may be regarded as a vector space of dimension $d+1$ by identifying a polynomial with its sequence of coefficients, i.e., the sequence $(c_0, c_1, c_2, \ldots, c_d)$ which is a vector in $\mathbf{R}^{d+1}$.

1. What formula from calculus expresses the value of c_j for $0 \leq j \leq d$ in terms of f?
2. What rules about derivatives imply that the function D from $\mathbf{R}^{d+1}$ to $\mathbf{R}^{d+1}$ given by the operation
$$f(X) \longmapsto f'(X)$$
is an abstractly linear function?
3. What is the kernel of D?
4. What is the image of D?
5. What is the matrix of D when it is expressed solely in terms of coefficients? *Hint:* Work it out for the special cases $d \;=\; 0, 1, 2,$ and $3,$ and then surmise a pattern.

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`http://math.albany.edu:8000/math/pers/hammond/course/mat220/assgt/la011026.html`